# EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

## T6: Vapour power cycles

*This is an extract from 'Real Life Examples in Thermodynamics: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2010 (ISBN:* 978-0-9842142-1-1*) and contains suggested exemplars within lesson plans for Sophomore Thermodynamics Courses. They were prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".*

## INTRODUCTION

(from *'Real Life Examples in Thermodynamics: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a sophomore level course in thermodynamics, increase the accessibility of the principles, and raise the appeal of the subject to students from diverse backgrounds. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. The 5E outline is not original and was developed by the Biological Sciences Curriculum Study[1] in the 1980s from work by Atkin & Karplus[2] in 1962. Today this approach is considered to form part of the constructivist learning theory[3].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations, common tables/charts, and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement and enhance. Similarly, it is anticipated that these lesson plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

This is the third in a series of such notes. The others are entitled 'Real Life Examples in Mechanics of Solids' (ISBN: 978-0-615-20394-2), 'Real Life Examples in Dynamics'(ISBN: 978-0-9842142-0-4).

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[1] Engleman, Laura (ed.), *The BSCS Story: A History of the Biological Sciences Curriculum Study*. Colorado Springs: BSCS, 2001.
[2] Atkin, J. M. and Karplus, R. (1962). Discovery or invention? *Science Teacher* 29(5): 45.
[3] e.g. Trowbridge, L.W., and Bybee, R.W., *Becoming a secondary school science teacher*. Merrill Pub. Co. Inc., 1990.

**POWER CYCLES**

6. Topic**: Vapour power cycles**

**Engage:**

Take a child's water pistol with a large reservoir into class together with a bucket and an electric kettle. If the windows in the room open, then perhaps you can fire the water pistol out of the window, otherwise use the bucket!

At the same time boil the kettle and produce clouds of steam. Perhaps you could ask a pair of students to hold down the kettle switch (until it boils dry) and to pump the water pistol (until it runs dry).

**Explore:**

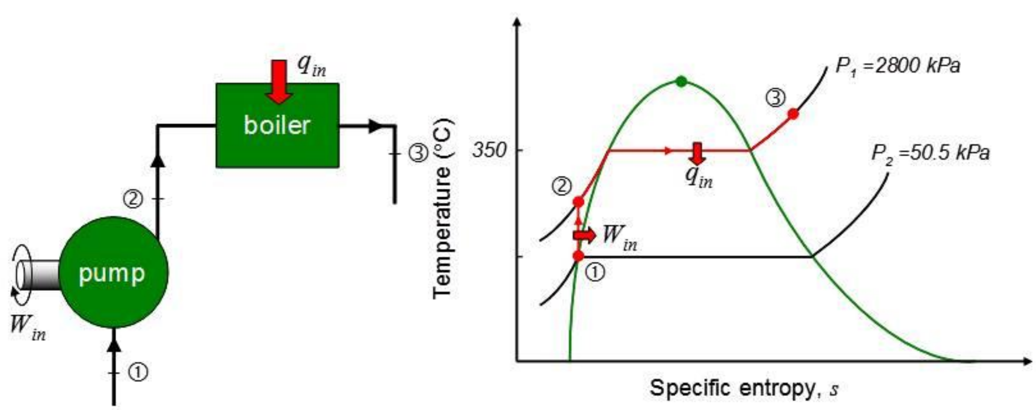
Ask the students to describe the water pistol and kettle in engineering terms, i.e. what type of devices are they?

Answers: a pump: '*a device that raises, transfers, delivers or compresses fluids*' and a boiler: *'a vessel used for boiling; the part of a steam generator in which water is converted into steam*' according to the Merriam-Webster Online Dictionary[4].

The water pistol and the kettle represent half of a vapor power cycle – the half in which we have to deliver work (via the pump) and heat transfer (via the boiler). You could pump some water (from the bucket) into the kettle with the water pistol to illustrate the relationship.

**Explain:**

We can draw the half of a Rankine cycle represented by the water pistol (pump) and kettle (boiler) as shown below and also plot it on a temperature-entropy plot.




[4] www.merriam-webster.com/dictionary/

For the typical temperature and pressure range illustrated in the T-s plot, the enthalpies can be determined from the steam tables (www.dofmaster.com/steam.html):

State 1: $P_1 = 50.5\,\text{kPa}$ and saturated water ($x = 0$) so

$$h_1 = h_f = 341.6059\,\text{kJ/kg},\ v_1 = v_f = 0.0010\,\text{m}^3\text{/kg and } s_1 = s_f = 1.0941\ \text{kJ/kg/C}.$$

State 2: $P_1 = 2800\,\text{kPa}$ and $s_2 = s_1 = 1.0941\ \text{kJ/kg/C}$

Work done between 1 and 2, $W_{in} = v_1(P_2 - P_1) = 0.0010(2800 - 50.5) = 2.7495\,\text{kJ/kg}$

Hence the work input is equal to the change in enthalpy using the first law of thermodynamics. Noting that enthalpy is the internal energy of a substance plus the work done against the ambient pressure. So,

$$W_{in} = h_2 - h_1$$

and $$h_2 = h_1 + W_{in} = 341.6059 + 2.7495 = 344.3554\,\text{kJ/kg}$$

State 3: $P_3 = P_2 = 2800\,\text{kPa}$ and $T_3 = 350\,°\text{C}$ so

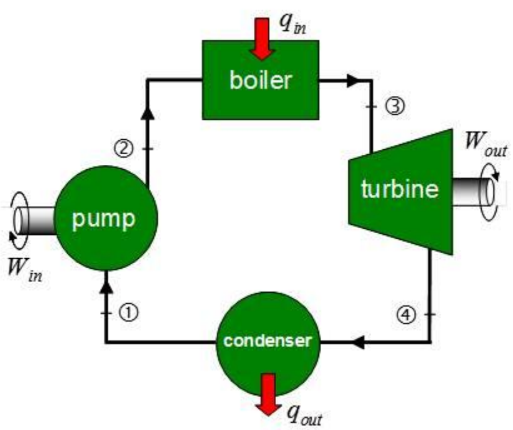
$$h_3 = 3121.8607\,\text{kJ/kg and } s_3 = 6.7842\,\text{kJ/kg/C}$$

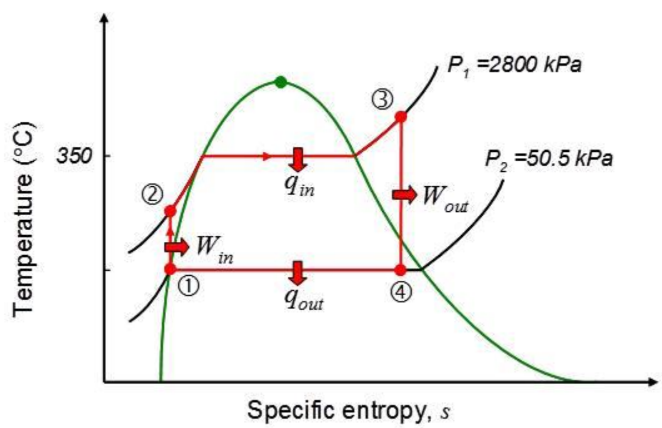
And, the heat supplied by the boiler is

$$q_{in} = h_3 - h_2 = 3121.8607 - 344.3554 = 2777.5053\,\text{kJ/kg}.$$

**Elaborate:**

Two further components are necessary to complete a closed vapor power cycle, i.e. a turbine and a condenser. We can add them to the schematic of the power plant and complete the plot of the cycle on the T-s diagram, as shown below.




State 4: $P_1 = P_4 = 50.5\,\text{kPa}$ and $s_4 = s_3 = 6.7842\,\text{kJ/kg/C}$ which corresponds to a saturated mixture. In order to find the quality of the mixture we need from the steam tables, $s_f = 1.0941$ kJ/kg/C and $s_{fg} = 6.4972\,\text{kJ/kg/C}$

thus $x = \frac{s_4 - s_f}{s_{fg}} = \frac{6.7842 - 1.0941}{6.4972} = 0.8758$ and

from the steam tables $h_f = 341.6059\,\text{kJ/kg}$ and $h_{fg} = 2304.7903\,\text{kJ/kg}$

so $h_4 = h_f + xh_{fg} = 341.6059 + (0.8758 \times 2304.7903) = 2360.0888\,\text{kJ/kg}$

Now, again using the first law of thermodynamics for the ideal process ($ds$ = 0)

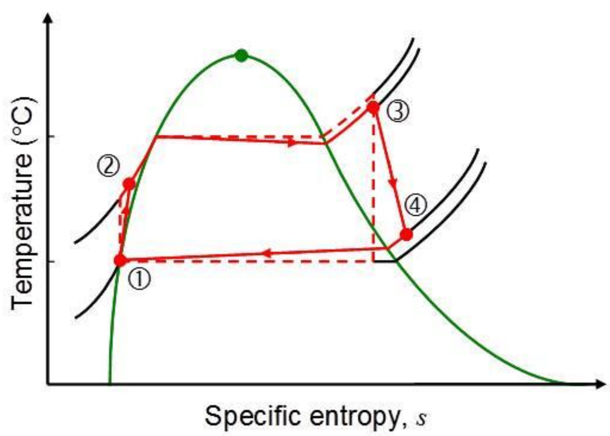
$$W_{out} = h_3 - h_4 = 3121.8607 - 2360.0888 = 761.7719\,\text{kJ/kg}$$

The efficiency is defined as the ratio of the net work and heat supplied, i.e.,

$$\eta_{th} = \frac{W_{out} - W_{in}}{q_{in}} = \frac{761.7719 - 2.7495}{2777.5053} = 0.2733 \text{ or } 27\%$$

This is the best efficiency that a power plant operating over this cycle can achieve because it does not involve any irreversibilities, i.e. the processes in the pump and turbine are isentropic (constant entropy) and there are no pressure losses in the boiler or condenser. We can draw an actual cyclic including these irreversibilities as shown below by the solid lines.**Elaborate:**

Two further components are necessary to complete a closed vapor power cycle, i.e. a turbine and a condenser. We can add them to the schematic of the power plant and complete the plot of the cycle on the T-s diagram, as shown below.




**Evaluate:**

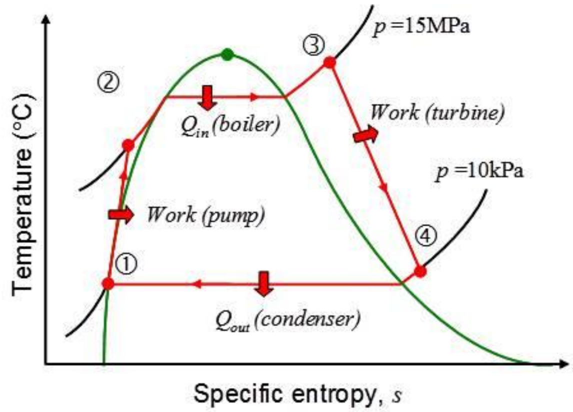
Invite students to attempt the following examples:

*Example 6.1*

At a particular location a hot spring is available to provide a heat source for a vapor power cycle of a power station and not far away a meltwater river from a glacier is available to provide a heat sink. The hot spring has a flow of 1500 litres/sec at 98°C and the meltwater river a flow of 5600 litres/sec at 1°C. Design a vapor power cycle for a 8MW power station using heat exchangers to input and remove heat from the working fluid and assuming an 80% isentropic efficiency in the turbine and pump with no pressure losses in the boiler or condenser. Attempt to minimize the environmental impact of the power plant.

*Example 6.2*

At a power station the 216kW feedwater pump supplies 12 kg/s of water to the boiler which in turn provides steam at 15MPa, 600°C to the turbine. The turbine provides 14.4MW of output and exhausts the steam at 10kPa. Calculate the efficiency of the power cycle assuming that the power station operates on a non-ideal vapor power cycle but that there are no pressure losses in the condenser or boiler.




Solution:

Work input via pump,

$$W_{in} = \frac{Power}{Flow \quad Rate} = \frac{216}{12} = 18\,\text{kJ/kg}$$

Work out from turbine,

$$W_{out} = \frac{Power}{Flow \quad Rate} = \frac{14400}{12} = 1200\,\text{kJ/kg}$$

Net. Work,

$W_{net} = W_{out} - W_{in} = 1200 - 18 = 1182\,\text{kJ/kg}$

State 3 (*input to turbine*): $P_3 = 15000\,\text{kPa}$, $T_3 = 600\,°\text{C}$, so using the steam tables $h_3 = 3579.7767$ kJ/kg and $s_3 = 6.6764\,\text{kJ/kg/C}$

State 4 (*output from turbine*): $P_4 = 10\,\text{kPa}$

Work output of turbine is known and can be equated to the change in enthalpy using the first law of thermodynamics

$$W_{out} = h_3 - h_4 \text{ so } h_4 = h_3 - W_{out} = 3579.7767 - 1200 = 2379.7767\,\text{kJ/kg}$$

Hence from the steam tables for $P_4 = 10\,\text{kPa}$ and $h_4 = 2379.7767\,\text{kJ/kg}$

$$T_4 = T_{sat} = 45.8328\,°\text{C},\ s_4 = 7.5084\,\text{kJ/kg/C}\ (> s_3) \text{ and } x = 0.9143,$$

so it is wet steam implying that ④ should have been drawn further to the left along the isobar where it becomes an isotherm.

State 1 (*inlet to pump*):

thus at the pump input/condenser outlet, $P_1 = P_4 = 10\,\text{kPa}$, $T_1 = T_4 = 45.8328\,°\text{C}$ and $x = 0$ so from the steam tables

$$h_1 = h_f = 191.8324\,\text{kJ/kg and } s_1 = s_f = 0.6493\,\text{kJ/kg/C}$$

Now, using the first law of thermodynamics for the work input by the pump

$$W_{in} = h_2 - h_1 \text{ so } h_2 = h_1 + W_{out} = 191.8324 + 18 = 209.8324\,\text{kJ/kg}$$

And for the boiler, $q_{in} = h_3 - h_2 = 3579.7767 - 209.8324 = 3369.94\,\text{kJ/kg}$

So the thermal efficiency, $\eta_{th} = \dfrac{w_{net}}{q_{in}} = \dfrac{1182}{3369.94} = 0.3507$ or 35%.