EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

S4: Compatibility & equilibrium

This is an extract from 'Real Life Examples in Mechanics of Solids: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2006 (ISBN:978-0-615-20394-2) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Solids Courses. Prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Mechanics of Solids: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a sophomore course in mechanics of solids, increase the accessibility of the principles and raise the appeal of the subject to students from a diverse background[1]. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. These are not original and were developed by the Biological Sciences Curriculum Study[2] in the 1980s from work by Atkin and Karplus[3] in 1962. Today they are considered to form part of the constructivist learning theory and a number of websites provide easy to follow explanations of them[4].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement. Similarly, it is anticipated that these lessons plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

Acknowledgements

Many of these examples have arisen through lively discussion in the consortium supported by the NSF grant (#0431756) on "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change" and the input of these colleagues is cheerfully acknowledged as is the support of NSF. The influence of the editor's mentors and peers at the University of Sheffield is substantial and is gratefully acknowledged since many of the ideas for these examples originate from tutorial questions developed and used in the Department of Mechanical Engineering in Sheffield over many years.

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[1] Patterson, E.A., Campbell, P.B., Busch-Vishniac, I., Guillaume, D.W., 2011, The effect of context on student engagement in engineering, *European J. Engng Education*, 36(3):211-224.
[2] http://www.bscs.org/library/BSCS_5E_Instructional_Approach_July_06.pdf
[3] Atkin, J. M. and Karplus, R. (1962). Discovery of invention? *Science Teacher* 29(5): 45.
[4] e.g. http://www.science.org.au/primaryconnections/constructivist.htm

STATICALLY INDETERMINATE PROBLEMS

4. Principle: **Compatibility & equilibrium**

Engage:

Bring your iPod into class dangling it from the earplug cable and also the front wheel of your bike again. Ask about the connection between them. They are both statically indeterminate systems – you need to consider both forces and displacements to find the stresses. Remind them how in the earplug cable, the wire and insulation share the load and must have the same extension or displacements. Similarly in the front wheel, the tire constrains the inner tube and they have the same radial displacement.

You might want to consider breaking the class into pairs, and asking them to draw the free-body diagrams for the iPod cable and insulation and then construct the equations of equilibrium and attempt to solve them. This will illustrate that the problem is indeterminate when only considering the principle of equilibrium.

Explore:

Highlight that redundancy is a feature of statically indeterminate systems. From a structure viewpoint we don't need the wire and the insulation. Of course we do for signal transmission. Similarly in the front wheel we don't need the tube and tire from a structural perspective, but the tire would leak and the tube would puncture so we need both for a safe and comfortable ride. This problem has been solved in cars…

Explain:

Redundancy results in there being insufficient information available to solve for internal stress using equilibrium of forces alone. Need to also consider compatibility of displacements for various elements in the structure.

For an iPod dangling from earplug cable:

Equilibrium of forces implies,

Weight of iPod, $mg = F_{wire} + F_{insulation}$ (i)

Compatibility of displacements requires extension of wire and insulation to be equal, i.e.

$\delta_{wire} = \delta_{insulation}$ (ii)

Elaborate:

So use the expressions above to solve for the same example as previously, i.e. a 30 gram iPod dangling from its earplug cable consisting of a copper wire of diameter 0.4mm with snugly fitting plastic insulation of external diameter 1mm:

$$\delta = \varepsilon L = \frac{\sigma L}{E}\frac{FL}{AE}$$

so from (ii)

$$\frac{F_{wire}L}{(\pi d^2/4)(E_{wire})} = \frac{F_{insulation}L}{(\pi/4)(d_o^2 - d_i^2)(E_{insulation})}$$

And $$F_{wire} = \frac{d^2 E_{wire} F_{insulation}}{(d_o^2 - d_i^2)(E_{insulation})} = \frac{d^2}{(d_o^2 - d_i^2)}\frac{E_{wire}}{E_{insulation}}(mg - F_{wire})$$

Substituting for the forces from (i) and taking $d = d_i$

$$F_{wire} = \frac{d^2 E_{wire} mg}{(d_o^2 - d_i^2)E_{insulation} + d^2 E_{wire}}$$

$$F_{wire} = \frac{(0.4^2)(110\times10^9)(0.03\times9.81)}{(1^2 - 0.4^2)(2\times10^9) + 0.4^2(110\times10^9)} = 0.268\,\text{N}$$

thus $$F_{insulation} = mg - F_{wire} = (0.03\times9.81) - 0.268 = 0.026N$$

and now $$\sigma_{wire} = \frac{F_{wire}}{(\pi/4)d^2} = \frac{0.268}{(\pi/4)\times0.4^2} = 2.13\,\text{N/mm}^2$$

$$\sigma_{insulation} = \frac{F_{insulation}}{(\pi/4)(d_o^2 - d_i^2)} = \frac{0.026}{(\pi/4)\times(1^2 - 0.4^2)} = 0.039\,\text{N/mm}^2$$

compare these values with those obtained when we considered the whole load to be borne by the wire or insulation, i.e. 2.34 N/mm^2 and 0.29 N/mm^2 respectively.

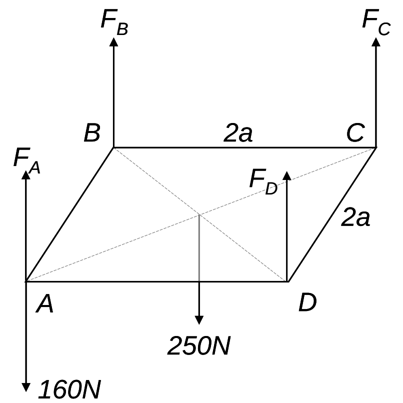
Evaluate:

Ask students to attempt the following examples:

Example 4.1

In the lobby of a natural history museum, a square horizontal plate made of polycarbonate is suspended at the corners by means of cables 6m long and 3mm diameter attached to the ceiling. The weight of the plate and the dinosaur skeleton displayed on it is 250N and the cables are adjusted so that the weight is evenly distributed between them. It is decided to add the skeleton of the dinosaur's prey attached directly below one corner. If the additional skeleton weighs 160N find the stress in each wire for the complete exhibit.

Solution



Using equilibrium of forces

Moments about BC: $250a + 2\times160a = 2a(F_A + F_D)$

Thus $285 = F_A + F_D$ (i)

Moments about DA: $250a = 2a(F_B + F_C)$

Thus $125 = F_B + F_C$ (ii)

Moments about CD: $250a + 2\times160a = 2a(F_A + F_C)$

Thus $285 = F_B + F_A$ (iii)

From (i) and (ii) $F_B = F_D$ (iv)

this can be deduced from the symmetry of the problem

Using compatibility of displacements

By similar triangles: $\delta_A - \delta_B = \delta_B - \delta_C$ (v)

All the wires are identical in geometry and material so $F \propto \delta$

And $F_A - F_B = F_B - F_C$ (vi)

Solving simultaneously

Substituting (iv) in (vi) $F_A + F_c = 2F_B$ (vii)

Substitute in (iii) $3F_A + F_c = 570$ (viii)

Substitute (ii) in (viii) $F_A + 3F_c = 250$ (ix)

Hence $F_A = 182.5\,\mathrm{N}$, $F_C = 22.5\,\mathrm{N}$, and $F_B = 102.5\,\mathrm{N}$

Now the cross-section area of a cable is $A = \dfrac{\pi(3\times10^{-3})^2}{4} = 7.07\times10^{-6}\,\mathrm{m}^2$

Thus,

$$\sigma_A = \frac{182.5}{7.07 \times 10^{-6}} = 25.8 \times 10^6 \text{ N/m}^2$$

$$\sigma_B = \frac{102.5}{7.07 \times 10^{-6}} = 14.5 \times 10^6 \text{ N/m}^2$$

$$\sigma_A = \frac{22.5}{7.07 \times 10^{-6}} = 3.18 \times 10^6 \text{ N/m}^2$$

The stresses in wires are 25.8, 14.5, 14.5 and 3.18MPa.

Example 4.2

Ask students to look for two other examples in their everyday life and explain how the above principles apply to each example.