EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

F9: Flow over bodies

This is an extract from 'Real Life Examples in Fluid Mechanics: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2011 (ISBN:978-0-9842142-3-5) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Fluids Courses. They were prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Fluid Mechanics: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a sophomore level course in fluid mechanics, increase the accessibility of the principles, and raise the appeal of the subject to students from diverse backgrounds. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. The 5E outline is not original and was developed by the Biological Sciences Curriculum Study[1] in the 1980s from work by Atkin & Karplus[2] in 1962. Today this approach is considered to form part of the constructivist learning theory[3].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations, common tables/charts, and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement and enhance. Similarly, it is anticipated that these lesson plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

It is assumed that students have acquired a knowledge and understanding the following topics: first and second law of thermodynamics, Newton's laws, free-body diagrams, and stresses in pressure vessels.

This is the fourth in a series of such notes. The others are entitled 'Real Life Examples in Mechanics of Solids', 'Real Life Examples in Dynamics' and 'Real Life Examples in Thermodynamics'. They are available on-line at www.engineeringexamples.org.

Eann A. Patterson
A.A. Griffith Chair of Structural Materials and Mechanics
School of Engineering, University of Liverpool, Liverpool, UK
& Royal Society Wolfson Research Merit Award Recipient

---

[1] Engleman, Laura (ed.), *The BSCS Story: A History of the Biological Sciences Curriculum Study*. Colorado Springs: BSCS, 2001.
[2] Atkin, J. M. and Karplus, R. (1962). Discovery or invention? *Science Teacher* 29(5): 45.
[3] e.g. Trowbridge, L.W., and Bybee, R.W., *Becoming a secondary school science teacher*. Merrill Pub. Co. Inc., 1990.

FLOW

9. <u>Topic</u>**: Flow over bodies (external flow)**

Engage:

Wear a swim cap into class – this is bound to engage the students' attention. Ask them why competition swimmers wear caps and suits [Answer: to reduce drag]. Ask students to estimate whether the flow associated with a swimmer is laminar or turbulent.

Explore:

Estimate the Reynolds number for a competitive swimmer. The world record for 50m freestyle is about 20s; so $v = 50/20 = 2.5$ m/s hence for a swimmer of height 1.8m

$$\mathrm{Re} = \frac{\rho v L}{\mu} = \frac{1000 \times 2.5 \times 1.8}{1 \times 10^{-3}} = 4.5 \times 10^{6}$$

So the flow is turbulent. You could show some video footage of freestyle swimmers from Youtube[4] by searching for video "*Total Immersion Swimming Freestyle Demo by Shinji Takeuchi*"[5]. To illustrate the separation of the boundary layer show again the video of separation around a tennis ball from lesson 4[6].

Explain:

Explain that swimmers experience two forms of drag: frictional or skin drag and pressure or form drag. Also that drag due to pressure is almost always greater than drag due to skin friction. Swimmers believe that their swim suit and cap decrease skin drag by reducing the boundary layer. This is important because an increase in turbulence in the boundary layer increases the eddies in the boundary layer, which consume energy reducing the kinetic energy and speed of the swimmer[7]. At slow speeds for a well-streamlined body the flow will be laminar and frictional resistance will dominate. At higher speeds, the boundary layer grows and pressure resistance increases. This occurs because the separation of the boundary layer from the body moves closer to the front further increasing the pressure drag but reducing the drag due to skin friction which gives an overall increase in total drag. The pressure resistance force is given by

$$F = C_D S_M \frac{\rho v^2}{2}$$

---

[4] www.youtube.com/watch?v=rJpFVvho0o4

[5] Alternate is http://www.youtube.com/watch?v=AInQMmn-0Nw&feature=related) or search for a video entitled: 'How to improve your freestyle'

[6] www.youtube.com/watch?v=7KKFtgx2anY

[7] Zatsiorsky, V.M., *Biomechanics in sport: performance enhancement and injury prevention*, IOC Medical Commission, International Federation of Sports Medicine.

where S_M is the maximal cross-sectional area interacting with the water and C_D is the drag coefficient. Typical values for a swimmer of C_D are 0.58 to 1.04 compared to between 0.05 and 0.08 for a dolphin[23]. A dolphin has a better streamlined body without the local pressure resistance centers formed by the head, shoulders, buttocks, knees, heels, etc in a human.

Elaborate:

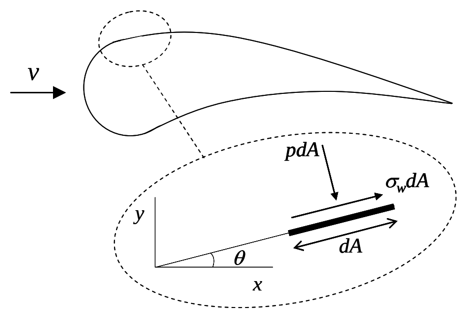
Explain that a body immersed in and moving relative to a fluid interacts with the fluid and experiences a resultant force in the direction of the upstream flow, known as drag, and a force normal to the flow, known as lift. A simple explanation of lift is given in a short video entitled '*The Magic of Airfoils*'[8] (search in Youtube using italicized words) you might also want to show a video of a basic student experiment, for example search on YouTube for '*Wind Tunnel Basic Airfoil Test*'[9].

It can be shown that

lift, $$L = \int dF_y = -\int (p\sin\theta)dA + \int (\tau_W \cos\theta)dA$$

and

drag, $$D = \int dF_x = \int (p\cos\theta)dA + \int (\tau_W \sin\theta)dA$$



So that lift and drag coefficients can be defined as

$$C_L = \frac{L}{\frac{1}{2}\rho v^2 A} \text{ and } C_D = \frac{D}{\frac{1}{2}\rho v^2 A}$$

Return to the tennis ball video and highlight that inviscid flow around it are symmetric about the *x*-axis and so the lift and drag are zero in these conditions. However, if the sphere is spinning about the *z*-axis, then the rotation will drag some fluid around so that the flow is no longer symmetric and both lift and drag are created.

Ask the students for examples of the application of this phenomenon [answer: curved balls, floaters and sinkers in baseball, hooking or slicing a golf ball, and curving a ball in soccer]. You could show the '*Best Curve in Soccer History*' search for this title on YouTube[10].

For horizontal flight, the lift generated by spinning of the ball must equal the weight of the ball, so for a soccer ball of mass 450g, diameter 22cm traveling at 30ms^{-1}

$$mg = L = \tfrac{1}{2}\rho_{H_20} v^2 A C_L$$

or $$C_L = \frac{mg}{\frac{1}{2}\rho_{H_2O} v^2 \times \frac{\pi}{4} d^2} = \frac{0.45 \times 9.81}{\frac{1}{2} \times 1.2 \times 30^2 \times \frac{\pi}{4} \times 0.22^2} = 0.215$$

Empirical data suggests that this can be achieved with

---

[8] www.youtube.com/watch?v=yp7w3p0P2tw
[9] www.youtube.com/watch?v=1PXJ3HhV2mE
[10] www.youtube.com/watch?v=-jLD_2ULFxc

$$\frac{d\omega}{2v} = 0.9$$

So $$\omega = \frac{0.9 \times 2v}{d} = \frac{0.9 \times 2 \times 30}{0.22} = 245\,\text{rad/s} \equiv 2343\text{ rpm}$$

It would be relevant to discuss that external flows, such as those discussed above become turbulent when the Reynolds Number, Re > 5×10^5 (this compares to fully turbulent flow at Re > 4000 and fully laminar flow at Re < 2300 for internal flows).

Evaluate:

Invite the students to attempt the following examples:

Example 9.1

Blood cells are damaged by turbulent flow. Estimate the maximum size of a device that could be introduced during surgery on the aortic valve without creating turbulent flow around it.

Solution:

The flow velocity through the aortic valve is typically 6m/s[11] and the density[12] and viscosity[13] of arterial blood is 1050kg/$m^3$and 4×10^{-3} Pa.s respectively. For laminar flow to be maintained Re < 1000 and

$$\text{Re} = \frac{\rho v L}{\mu}$$

so $$L = \frac{\text{Re}\,\mu}{\rho v} < \frac{1000 \times 4 \times 10^{-3}}{1050 \times 6} = 6.35 \times 10^{-4}\text{ m or 0.6mm}$$

Example 9.2:

When you take a job delivering pizzas you are expected to fit a sign 0.8m × 0.3m to the top of your car. If you drive at an average of 30mph (≡ 13.41ms^{-1}), calculate the added cost of fuel used per hour. Assume the car is 30% efficient. The drag coefficient for a long flat plate is 1.98[14]. If you lived in area with some long empty roads so that your average speed increased to 45mph, what would be the cost of the extra fuel?

Solution:

The definition of the drag coefficient is

---

[11] Mohiaddin, RH, Firmin, DN, Longmore, DB., Age-related changes of human aortic flow wave velocity measured non-invasively by magmetic-resonance imaging, J. Appl. Physiology, 74(1):492-492, 1993.
[12] Kenner, T., The measurement of blood density and its meaning, Basic Res Cardiology, 84:111-124, 1989.
[13] Dormandy, JA, Influence of blood viscosity on blood flow and the effect of low molecular weight dextran, British Medical J., 4:716-719, 1971.
[14] See for example: http://www.engineeringtoolbox.com/drag-coefficient-d_627.html

$$C_D = \frac{D}{\frac{1}{2}\rho v^2 A}$$

so, the force (=drag) required to move the sign is

$$D = \tfrac{1}{2}\rho_{air} v^2 A C_D = \tfrac{1}{2} \times 1.2 \times 13.41^2 \times (0.8 \times 0.3) \times 1.98 = 51.3\,\mathrm{N}$$

Thus, the power required is

$$P = Fv = 51.3 \times 13.41 = 687\,\mathrm{W}$$

This needs to be generated by the combustion of fuel in the engine

$$Q = \frac{P}{\eta E} = \frac{687}{0.3 \times 32 \times 10^9} = 7.16 \times 10^{-8}\,\mathrm{m^3/s}$$

where E (J/kg) is the energy content of gasoline (≈ 32MJ/liter[15] ≡ 32,000MJ/m^3). And this quantity is equal to 0.068 gallons/hr or 24cents/hr if the price of gas is \$3.50/gallon (as it was at the time of writing in Michigan).

For an average speed of 45mph (≡20m/s), D = 114N, P=2280, Q=2.375×10-7 and the cost is 79 cents/hr; so a 50% increase in average speed costs three times as much.

[15] http://bioenergy.ornl.gov/papers/misc/energy_conv.html