EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

F8: Viscous pipe flow

This is an extract from 'Real Life Examples in Fluid Mechanics: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2011 (ISBN:978-0-9842142-3-5) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Fluids Courses. They were prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Fluid Mechanics: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a sophomore level course in fluid mechanics, increase the accessibility of the principles, and raise the appeal of the subject to students from diverse backgrounds. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. The 5E outline is not original and was developed by the Biological Sciences Curriculum Study[1] in the 1980s from work by Atkin & Karplus[2] in 1962. Today this approach is considered to form part of the constructivist learning theory[3].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations, common tables/charts, and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement and enhance. Similarly, it is anticipated that these lesson plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

It is assumed that students have acquired a knowledge and understanding the following topics: first and second law of thermodynamics, Newton's laws, free-body diagrams, and stresses in pressure vessels.

This is the fourth in a series of such notes. The others are entitled 'Real Life Examples in Mechanics of Solids', 'Real Life Examples in Dynamics' and 'Real Life Examples in Thermodynamics'. They are available on-line at www.engineeringexamples.org.

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[1] Engleman, Laura (ed.), *The BSCS Story: A History of the Biological Sciences Curriculum Study*. Colorado Springs: BSCS, 2001.
[2] Atkin, J. M. and Karplus, R. (1962). Discovery or invention? *Science Teacher* 29(5): 45.
[3] e.g. Trowbridge, L.W., and Bybee, R.W., *Becoming a secondary school science teacher*. Merrill Pub. Co. Inc., 1990.

FLOW

8. Topic: **Viscous flow in pipes (internal flow)**

Engage:

Take a vacuum cleaner into the class and clean up after the previous class using one of the tools attached to the pipe. It should attract the attention of the students! Ask them to vote on whether the flow in the pipe is laminar or turbulent. You can use a YouTube video[4] to illustrate '*laminar flow in pipe*' and a corresponding video[5]: '*turbulent flow in a pipe*' by searching with the words in italics.

Explore:

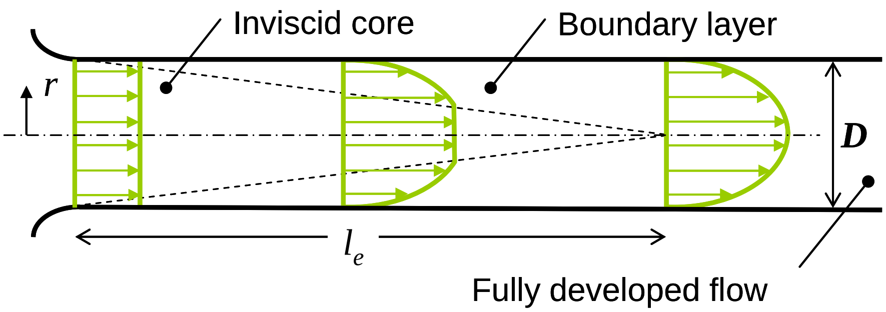
Estimate the Reynolds number of the vacuum cleaner pipe:

$$\mathrm{Re} = \frac{\rho_{air} v D}{\mu_{air}}$$

The density and viscosity of air at 20°C are 1.2 kg.m^{-3} and 19.8×10^{-6} Pa.s respectively. if the radius of the pipe is approximately 15mm and 150CFM (≡0.0706 m^{3}s^{-1}) is a typical airflow capacity. Thus

$$\mathrm{Re} = \frac{1.2 \times \left(\dfrac{0.0706}{\pi \times 0.015^2}\right) \times 0.03}{19.8 \times 10^{-6}} = 181{,}597$$

Fully turbulent flow occurs with Re>2,300 so those that voted for turbulent flow were correct.



Explain:

Explain how with no tool fitted to the pipe of the vacuum, the air enters with a nearly uniform velocity across the section. As the air moves along the pipe, viscous effects cause it to adhere to

[4] www.youtube.com/watch?v=KqqtOb30jWs
[5] www.youtube.com/watch?v=NplrDarMDF8&NR=1

the pipe wall creating a boundary layer in which viscous effects dominate. In the central section, beyond the boundary layer viscous effects are negligible; however, with distance along the pipe the boundary layer grows and eventually occupies the whole cross-section. The non-dimensional entrance length, l_e/D correlates well with Reynolds number.

Elaborate:

Explain that for laminar pipe-flow it can be shown that the velocity profile is described by

$$u(r)=\left(\frac{\Delta p D^2}{16\mu l}\right)\left[1-\left(\frac{2r}{D}\right)^2\right]$$

where the first term is the center-line velocity of the fluid. This expression describes a parabolic distribution. By integrating over the cross-section of the pipe the flowrate, Q, can be obtained as

$$Q=\frac{\Delta p \pi D^4}{128\mu l}$$

which is known as Poiseuille's law.

The viscous effects at the pipe wall are responsible for energy losses, which are known as head losses, h_L. It can be shown that

$$h_L=\frac{4l\tau_w}{\gamma D}$$

where τ_W is the wall shear stress. The resultant pressure drop is given by

$$\Delta p=\frac{8\mu v l}{r^2}$$

where r is the radius of the pipe.

In turbulent flows the velocity approaches a uniform distribution across the pipe with only a small viscous sub-layer close to the pipe wall. A number of empirical expressions exist to describe the shear stress and the velocity profile. A reasonable approximation is obtained using the inviscid Bernoulli equation and assuming a uniform velocity profile.

Losses in turbulent flow are assumed to arise from viscous effects in straight pipes and are known as major losses, $h_{L_{major}}$: and those from head losses in pipe components, known as minor losses, $h_{L_{minor}}$

By non-dimensional analysis, it can be shown that the pressure drop along a pipe containing turbulent flow is given by

$$\frac{\Delta p}{\frac{1}{2}\rho v^2}=\phi\left(\frac{\rho v D}{\mu},\frac{l}{D},\frac{\varepsilon}{D}\right)$$

where ε is a measure of the roughness of the pipe wall. If we assume the pressure drop is proportional to pipe length, which is supported by experimental evidence, then

$$\Delta p = f\frac{\rho v^2}{2}\frac{l}{D}$$

where *f* is the friction factor and

$$f = \phi\left(\mathrm{Re}, \frac{\varepsilon}{D}\right)$$

And, by substitution in the SFEE, it can be shown that

$$h_{L_{major}} = f\frac{l}{D}\frac{v^2}{2g}$$

For minor losses a loss coefficient, K_L, is usually defined such that

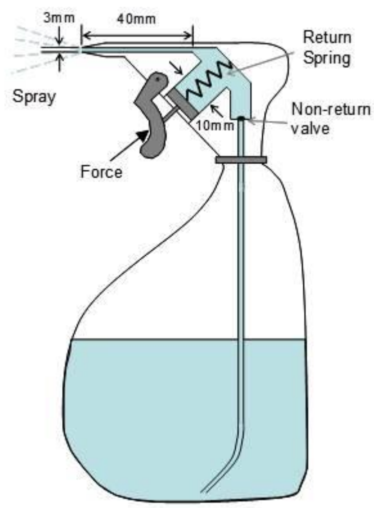
$$K_L = \frac{h_{L_{minor}}}{\left(v^2/2g\right)} = \frac{\Delta p}{\frac{1}{2}\rho v^2}$$

Finally for the vacuum tube we can estimate the head losses as follows: for a plastic tube the relative roughness can be taken as zero, and so using a Moody diagram[6] for a Reynolds number of 1.8×10^5 we find a friction factor of about 0.0155, hence the pressure drop due to major losses is

$$\Delta p = f\frac{\rho v^2}{2}\frac{l}{D} = 0.0155\frac{1.2\times\left(\frac{0.0706}{\pi\times0.015^2}\right)^2}{2}\frac{1}{0.03} = 3092\,\mathrm{Pa/m}$$

And for minor losses assuming a 90° flanged elbow in the pipe for which K_L=0.3[7]

$$\Delta p = \tfrac{1}{2}\rho v^2 K_L = \frac{1.2\times\left(\frac{0.0706}{\pi\times0.015^2}\right)^2}{2}\times0.3 = 1796\mathrm{Pa.}$$



Evaluate:

Ask the students to attempt the following examples:

Example 8.1

Investigate the mechanism in a spray bottle and identify the size and shape of the fluid pipes though which the liquid is drawn up and then squirted. Estimate whether the flow will be turbulent or laminar and then calculate the pressure drop due to head losses during squirting.

6 www.ecourses.ou.edu/cgi-bin/view_anime.cgi?file=d06123.swf&course=fl&chap_sec=06.1
7 For example: www.engineeringtoolbox.com/minor-loss-coefficients-pipes-d_626.html

<u>Solution:</u>

We need to estimate the Reynold's number,

$$\mathrm{Re} = \frac{\rho_{H_2O} v D}{\mu}$$

for the pipe to the spray nozzle, $D \approx$3mm and assuming we are spraying water so ρ = 1000 kg/m^3 and μ = 1×10^{-3} Pa.s. Let's say it takes 250 squirts to empty an 8oz (236,584mm^3) spray bottle then it dispenses 946mm^3 (=236,584/250) at each squirt. You can check this by estimating the stroke volume for the piston; if the piston is of radius 5mm then it would require a stroke of 12mm (=946/($\pi\times5^2$)) which seems reasonable. If a squirt takes about 1 second, then the velocity will be 134mm/s (=946/($\pi\times1.5^2$)). So substituting for the Reynolds number

$$\mathrm{Re} = \frac{1000\times0.134\times0.003}{1\times10^{-3}} = 402$$

This is sufficiently low for the flow to remain laminar. The return of the piston is generally slower so the flow will also be laminar.

Consequently the pressure drop can be estimated from

$$\Delta p = \frac{8\mu_{H_2O} v l}{r^2} = \frac{8\times0.001\times0.134\times0.04}{0.0015^2} = 19\,\mathrm{Pa}$$

This pressure plus the spring stiffness equates to the force needed to squirt the liquid.

<u>Example 8.2</u>

Estimate how hard you would need to pump to induce turbulent flow in the tube connecting a hand-pump to your bicycle tire. For these circumstances estimate the pressure drop due to major and minor losses.

<u>Solution:</u>

For the onset of turbulence a Reynolds number of more than 2300 is needed and for fully developed turbulent flow Re>10,000.

$$\mathrm{Re} = \frac{\rho v D}{\mu}$$

Everything is fixed except the velocity of the air, which is controlled by your pumping, so for a tube of internal diameter 3mm

$$v = \frac{\mathrm{Re}\,\mu}{\rho D} = \frac{2300\times19.8\times10^{-6}}{1.2\times0.003} = 12.65\,\mathrm{m/s}$$

which requires a volume flow rate, $Q = vA = 12.65\times\frac{\pi\times0.003^2}{4} = 8.94\times10^{-5}\,\mathrm{m^3/s}$

If the internal diameter of the pump is 30mm, then you will need to move the piston at

$$v = \frac{Q}{A_{piston}} = \frac{8.94\times10^{-5}}{\frac{\pi}{4}\times0.03^2} = 0.1265\text{m/s}$$

The pressure drop resulting from major losses can be estimated using

$$\Delta p = f\,\frac{\rho v^2}{2}\frac{l}{D}$$

For a plastic tube it can be assumed to be smooth, and from a Moody chart for Re=2300, the friction factor, *f* = 0.03. Hence for a pipe of length 0.125m

$$\Delta p = f\,\frac{\rho v^2}{2}\frac{l}{D} = 0.03\times\frac{1.2\times0.1265^2}{2}\frac{0.125}{0.003} = 0.012\,\text{Pa}$$

The only significant minor losses are in the valve and a typical loss coefficient is $K_L = 2$, so

$$\Delta p = \tfrac{1}{2}\rho v^2 K_L = \frac{1.2\times0.1265^2}{2}\times2 = 0.0192\text{Pa}$$

Thus, in this case, the head loss due to the minor losses is greater than that due to major losses occurring from viscous effects, which is actually more common.