EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

F11: Compressible flow

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This is an extract from 'Real Life Examples in Fluid Mechanics: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2011 (ISBN:978-0-9842142-3-5) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Fluids Courses. They were prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Fluid Mechanics: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a sophomore level course in fluid mechanics, increase the accessibility of the principles, and raise the appeal of the subject to students from diverse backgrounds. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. The 5E outline is not original and was developed by the Biological Sciences Curriculum Study[1] in the 1980s from work by Atkin & Karplus[2] in 1962. Today this approach is considered to form part of the constructivist learning theory[3].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations, common tables/charts, and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement and enhance. Similarly, it is anticipated that these lesson plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

It is assumed that students have acquired a knowledge and understanding the following topics: first and second law of thermodynamics, Newton's laws, free-body diagrams, and stresses in pressure vessels.

This is the fourth in a series of such notes. The others are entitled 'Real Life Examples in Mechanics of Solids', 'Real Life Examples in Dynamics' and 'Real Life Examples in Thermodynamics'. They are available on-line at www.engineeringexamples.org.

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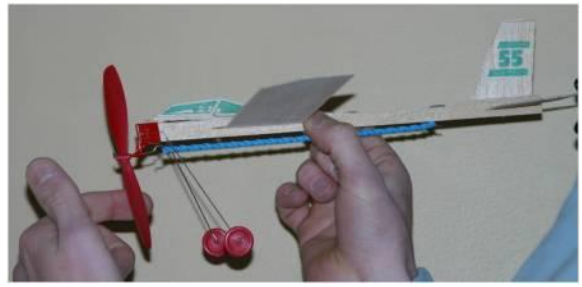
[1] Engleman, Laura (ed.), *The BSCS Story: A History of the Biological Sciences Curriculum Study*. Colorado Springs: BSCS, 2001.
[2] Atkin, J. M. and Karplus, R. (1962). Discovery or invention? *Science Teacher* 29(5): 45.
[3] e.g. Trowbridge, L.W., and Bybee, R.W., *Becoming a secondary school science teacher*. Merrill Pub. Co. Inc., 1990.

APPLICATIONS

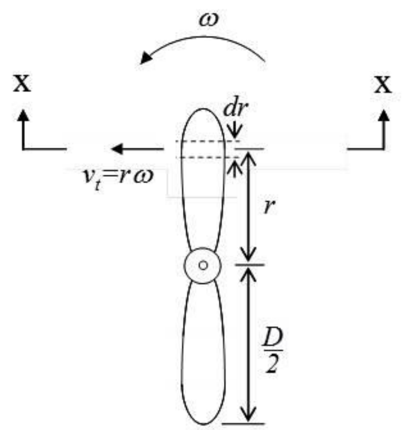
12. Topic: **Turbomachines**

Engage:

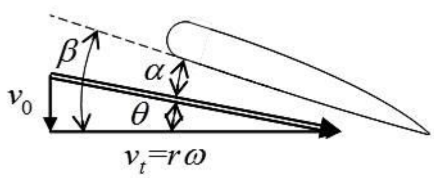


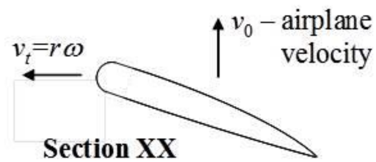
Take a pack or two of '*Propeller Balsa-Wood Model Planes*' into class and share them around the students[4]. Invite the students, working in pairs, to put them together, wind up the propeller and let the planes fly around the room.

Explore:



When they have had some fun, ask them to work on a velocity diagram for an element of the blade and determine the velocity of the air relative to the blade. Probably, you will have to get them started by drawing the sketch opposite and the section XX. Invite a pair to draw their velocity diagram for the rest of the class:





The angle of the airflow is given by

$$\theta = \tan^{-1}\frac{v_0}{r\omega}$$

and its speed is given by $r\omega/\cos\theta$. The angle, β is known as the pitch angle and the difference between θ and β is the local angle of attack, $\alpha(=\beta-\theta)$.

Explain:

Tell the students that it can be shown by dimensional analysis that

$$\frac{F_T}{\rho\omega^2 D^4} = \phi\left(\frac{v_0}{\omega D}, \frac{\rho D^2\omega}{\mu}\right)$$

[4] At the time of writing a pack of 12 were available from Amazon.com for $12.99. Try searching Amazon.com using the words in italics above.

where F_T is the propeller thrust, D is the propeller diameter, ω is the rotational speed, v_0 is the plane velocity, and ρ and μ are the air density and viscosity respectively. The Π-group on the left is known as the thrust coefficient, C_T and the first one on the right as the advance ratio. In most applications the Reynolds number is high and so the thrust coefficient is largely independent of Reynolds number, so

$$C_T \propto \frac{v_0}{\omega D}$$

The advance ratio is related to the relative velocity of the air at the propeller tip by

$$\theta = \tan^{-1}\frac{2v_0}{\omega D}$$

A similar analysis can be performed for the power, P, in which case everything is unchanged except the Π-group on the left becomes the power coefficient,

$$C_p = \frac{P}{\rho n^3 D^5}$$

Elaborate:

Discuss how pumps are characterized and selected. An axial flow pump or fan is essentially a propeller and the thrust coefficient can be redefined in terms of the pressure change, Δp or head difference, ΔH, since

$$F_T = \Delta p A = \gamma \Delta H A$$

and so

$$C_T = \frac{\gamma \Delta H A}{\rho \omega^2 D^4} = \frac{\pi g \Delta H}{4\omega^2 D^2}$$

but it is more usual to use the head coefficient,

$$C_H = \frac{4}{\pi}C_T = \frac{g\Delta H}{\omega^2 D^2}\ .$$

A discharge coefficient can be found by dimensional analysis to be $C_Q = \dfrac{Q}{\omega D^3}$

Axial-flow pumps are better suited to applications involving high discharge, Q and low head, ΔH; while radial-flow pumps, such as centrifugal pumps, are more appropriate for low discharge, high head applications. A Π-group known as the specific speed, n_s is used when selecting the best pump for an application from a range of geometrically similar pumps.

$$n_S = \frac{C_Q^{\frac{1}{2}}}{C_H^{\frac{3}{4}}} = \frac{\omega Q^{\frac{1}{2}}}{(g\Delta H)^{\frac{3}{4}}}$$

(Note: in the US it is common to use $N_S = NQ^{\frac{1}{2}} \big/ \Delta H^{\frac{3}{4}}$, which is not non-dimensional, since N is in revolutions per minute, Q in gallons per minute and ΔH in feet. The specific speed is closely related to the susceptibility to cavitation on the suction side. Cavitation should not occur, otherwise damage may be incurred in the blades of the pump. Usually, the specific speed is modified by replacing the pressure head by the difference between the pressure on the suction side of the pump and the vapor pressure of the liquid being pumped, which is called the net positive suction head, NPSH, so

$$n_{SS} = \frac{\omega Q^{\frac{1}{2}}}{g^{\frac{3}{4}} \left(NPSH\right)^{\frac{3}{4}}}$$

where n_{SS} is known as the suction specific speed and in the US is modified to $N_{SS} = NQ^{\frac{1}{2}} \big/ NPSH^{\frac{3}{4}}$ and a critical value of N_{SS} is 8500.

Evaluate:

Invite students to attempt the following examples:

Example 12.1

The water pump in a car is of the centrifugal type and sucks water from the radiator at about 15psi and 140°F before pumping it into the engine block. If the coolant flow rate required is 100 gallons per minute, calculate the maximum speed the pump can run at to avoid cavitation.

Solution:

The pressure head, p in psi can be converted to the pressure head, h in feet using

$$h = \frac{2.31 \times p}{\gamma_{SG}}$$

where γ_{SG} is the specific gravity or relative density of the fluid. Thus, for the radiator

$$h = \frac{2.31 \times p}{\gamma_{SG}} = \frac{2.31 \times 15}{1} = 34.7 \text{ ft}$$

Vapor pressure of water at 140°F is 2.9psi[5] which can also be converted to give

$$\frac{2.31 \times 2.9}{1} = 6.7 \text{ ft}$$

So the Net Positive Suction Head, NPSH $= 34.7 - 6.7 = 28 \text{ ft}$

$$N_{SS} = NQ^{\frac{1}{2}} \big/ NPSH^{\frac{3}{4}} < 8500$$

thus $$N < \frac{8500 \times NPSH^{\frac{3}{4}}}{Q^{\frac{1}{2}}} = \frac{8500 \times 28^{\frac{3}{4}}}{100^{\frac{1}{2}}} = 10{,}300 \text{rpm}$$

[5] From for example http://www.engineeringtoolbox.com/water-vapor-saturation-pressure-air-d_689.html

Example 12.2

If the tip speed of a propeller a 2m diameter is not too exceed a Mach number of 0.5 in ambient conditions, calculate the maximum speed of rotation. Then if the local angle of attack is to be zero for a forward velocity of 200mph, find the pitch angle as a function of distance along the blade from the tip.

Solution:

$$M_a = \frac{v}{c} = \frac{v}{\sqrt{kRT}} \quad \text{so} \quad \hat{v}_t = M_a\sqrt{kRT}$$

and $\quad v_t = r\omega \quad$ so $\quad \hat{v}_t = \dfrac{D\omega}{2}$

equating $M_a\sqrt{kRT} = \dfrac{D\omega}{2}$

and $\quad \omega = \dfrac{2M_a\sqrt{kRT}}{D} = \dfrac{2\times 0.5\times\sqrt{1.4\times 287\times 293}}{2} = 171$rad/s or 1640 rpm (=171×60/2π).

For local angle of attack, $\alpha = 0 = (\beta - \theta)$ so $\beta = \theta = \tan^{-1}\dfrac{v_0}{r\omega} = \tan^{-1}\dfrac{89}{r\times 171} \cong \tan^{-1}\dfrac{1}{2r}$ or about 30° at the tip and approaches 80° towards the hub (r = 0.1m) at the center.