EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

F10: Open channel flow

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This is an extract from 'Real Life Examples in Fluid Mechanics: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2011 (ISBN:978-0-9842142-3-5) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Fluids Courses. They were prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Fluid Mechanics: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a sophomore level course in fluid mechanics, increase the accessibility of the principles, and raise the appeal of the subject to students from diverse backgrounds. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. The 5E outline is not original and was developed by the Biological Sciences Curriculum Study[1] in the 1980s from work by Atkin & Karplus[2] in 1962. Today this approach is considered to form part of the constructivist learning theory[3].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations, common tables/charts, and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement and enhance. Similarly, it is anticipated that these lesson plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

It is assumed that students have acquired a knowledge and understanding the following topics: first and second law of thermodynamics, Newton's laws, free-body diagrams, and stresses in pressure vessels.

This is the fourth in a series of such notes. The others are entitled 'Real Life Examples in Mechanics of Solids', 'Real Life Examples in Dynamics' and 'Real Life Examples in Thermodynamics'. They are available on-line at www.engineeringexamples.org.

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[1] Engleman, Laura (ed.), *The BSCS Story: A History of the Biological Sciences Curriculum Study*. Colorado Springs: BSCS, 2001.

[2] Atkin, J. M. and Karplus, R. (1962). Discovery or invention? *Science Teacher* 29(5): 45.

[3] e.g. Trowbridge, L.W., and Bybee, R.W., *Becoming a secondary school science teacher*. Merrill Pub. Co. Inc., 1990.

FLOW

10.   Topic**: Flow in open channels**

Engage:

Show a video clip of the ultimate waterslide by searching on YouTube[4] for '*Barclay card advert-waterslide 2008 [HQ]*'. Ask students, working in pairs, to identify the two essential differences between open channel flow and flow in a pipe.

Explore:

Invite a couple of pairs to present their conclusions. Discuss the obvious one that the presence of the free surface and highlight the perhaps less obvious one that the flow is driven by gravity. It cannot be driven by a pressure difference (e.g. from pump) because of the negligible inertial and viscous effects of the atmosphere above the liquid.

The free surface of the fluid in the channel allows waves to exist and the Froude number characterizes their motion. For instance when a stone is thrown into a slow flowing canal the resultant waves ripple out more or less equally in all directions, including upstream. This is termed sub-critical flow for which flow velocity, v < wave velocity, c. However, when a stone is thrown into a fast flow river, waves only propagate downstream because $v>c$ and the flow is termed supercritical. The ratio of the flow and wave speed is the Froude number

$$\text{Fr} = \frac{v}{(gy)^2} = \frac{v}{c}$$

where y is the mean depth and g is gravitational acceleration.

The behavior of subcritical (Fr<1), critical (Fr=1), and supercritical (Fr>1) flow can be very different.

We have probably all thrown a stone in a river but there is videoclip of '*Yosemite National Park water ripples in slow motion*', which can be found by searching on Youtube[6] using the title.

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[4] www.youtube.com/watch?v=1WlRcXIO5ik
[5] www.flickr.com/photos/58017169@N06/5360523177/in/photostream/
[6] www.youtube.com/watch?v=T0tGXxF15-I

Explain:

Explain the flow down a water slide. For uniform flow in the slide the gravitational forces would have to equal the frictional and viscous forces associated with the walls of the slide that are dependent on the wall shear stress, τ_W, which is proportional to ρv^2, roughness, and Reynolds number. Reynolds number for open channel flow is defined as

$$\text{Re} = \frac{\rho v R_h}{\mu}$$

where R_h is the hydraulic radius of the channel, defined as the ratio of the cross-sectional area of the flow to the wetted perimeter. Generally, open channel flow is laminar when $\text{Re} < 500$ and turbulent when $\text{Re} > 12,500$.

At the entrance to the water slide when the velocity is low, the frictional forces will be smaller and the fluid will accelerate under the larger gravitational force until a terminal velocity is reached when the gravitational and frictional forces are equal.
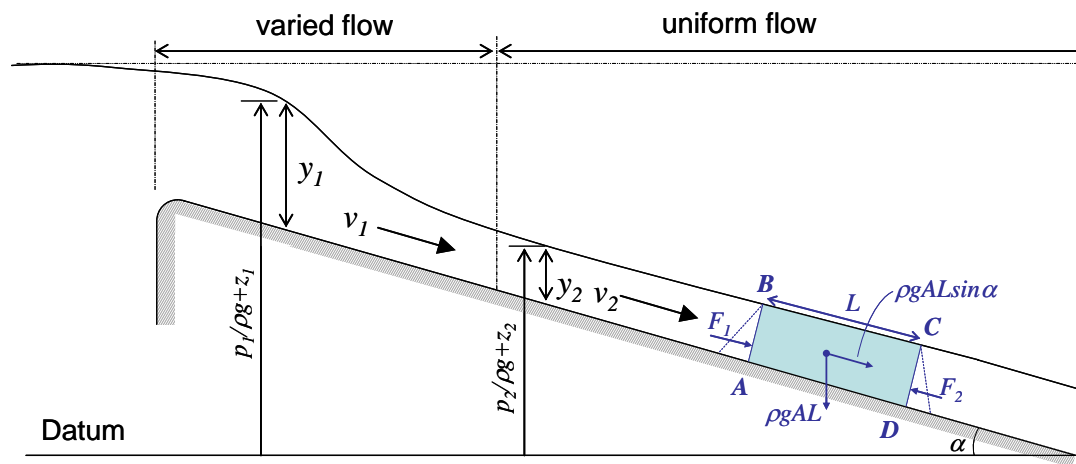
The requirement of continuity in a slide of constant cross-section implies that the depth of water must decrease as the velocity increases. Thus the entry to the slide is characterized by varied flow (changing depth and velocity) prior to the uniform flow region. In the uniform flow region, applying the momentum equation we have

$$F_1 - F_2 + \rho g A L \sin\alpha - \tau_W P L = 0$$

where F_1 and F_2 are the hydrostatic forces acting on the end planes of the section ABCD, and P is the wetted perimeter. If the velocity is constant, then based on continuity so is the depth and assuming the pressure is given by $\rho g z$ where z is depth below the surface, so the mean pressure on AB and CD is $\rho g z / 2$, and hence

$$F_1 = F_2 = \frac{\rho g y A}{2}$$

and for small slopes ($\alpha = \sin\alpha$) the momentum equation reduces to

$$\rho g \alpha = \tau_w \frac{P}{A} = \frac{\tau_w}{m}$$

where $m \, (= A/P)$ is known as the mean hydraulic depth. By definition, the friction factor,

$$f = \frac{\tau_w}{\frac{1}{2}\rho v^2}$$

so $\quad \rho g \alpha = \frac{f\rho v^2}{2m}$ and $v = \sqrt{\frac{2g}{f} m\alpha} = C\sqrt{m\alpha}$

where C is known as the Chezy coefficient (after Antoine Chezy (1718-1798) and is dependent on the friction factor. A civil engineer, Robert Manning (1816-1897) found that the coefficient was also dependent on the hydraulic mean depth so he proposed the empirical relationship

$$C = Mm^{\frac{1}{6}}$$

where M is known as the Manning constant. Note that in some US texts the Manning constant is defined as $n = M^{-1}$ and that M has basic units of $[L^3T^{-1}]$ so it important to take care when using it. Note that M=1.486/n where the units on left side are $[m^{1/3}s^{-1}]$ and on the right, feet and seconds.

Now, applying this analysis to a water slide with an approximately rectangular cross-section of width, 0.4m in which we wish to maintain a maximum water depth of 2cm. For plastic pipe, n =0.01[7] and

$$\text{Flow cross-section area} \qquad = \text{width} \times \text{depth} = 0.4 \times 0.002 = 0.0008\text{m}^2$$

$$\text{Wetted perimeter} \qquad = \text{width} + (\,2\times \text{depth}) = 0.4 + 0.004 = 0.404\text{m}$$

And, mean hydraulic depth, $\quad m = \dfrac{A}{P} = \dfrac{0.0008}{0.404} = 0.00198\text{m}$

Now combining the Chezy and Manning equations to give

$$v = n^{-1}m^{\frac{2}{3}}\alpha^{\frac{1}{2}}$$

So $\quad Q = Av = An^{-1}m^{\frac{2}{3}}\alpha^{\frac{1}{2}} \equiv AMm^{\frac{2}{3}}\alpha^{\frac{1}{2}}$

And assuming an angle of slope for the slide of about 20 degrees ($\alpha \approx 0.35$ rads), then

$$Q = AMm^{\frac{2}{3}}\alpha^{\frac{1}{2}} = 0.0008 \times 148.6 \times 0.00198^{\frac{2}{3}} \times 0.35^{\frac{1}{2}} = 1.1 \times 10^{-3} \text{ or about 66 liters/min.}$$

Finally, it is possible to generate discontinuities in open channel flow with or without a change in the channel geometry. A hydraulic jump involves a change in depth from y_1 to y_2 such that

$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8Fr_1^2}\right) \text{ for } \frac{y_2}{y_1} \geq 1 \text{ and } Fr_1 \geq 1.$$

[7] www.engineeringtoolbox.com/mannings-roughness-d_799.html

You can show a video of this occurring without a change in channel geometry by searching in YouTube for '*Water Engineering II S1 - Hydraulic Jump Practical*'[8] or with a geometry change by searching in YouTube for '*Hydraulic jump over weir*'[9].

Evaluate:

Invite the students to attempt the following examples:

Example 10.1

During a rain storm water flows along the edge of road. The cross-section of the flow forms a triangle with a base of length 100mm on the surface and base to apex depth of 15mm. If the road is concrete with a gradient of 1 in 25 in the direction of the flow (and traffic), calculate the discharge rate along the curbside gutter.
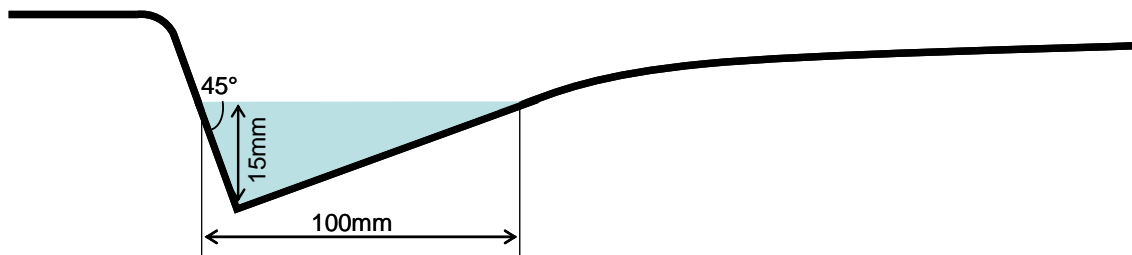
Solution:

Cross-section area of flow, $\qquad A = \frac{1}{2}bh = \dfrac{0.10 \times 0.015}{2} = 7.5 \times 10^{-4} \, \text{m}^3$

Wetted perimeter, $\qquad\qquad P = (21.2 + 80.2) \times 10^{-3} = 0.101 \, \text{m}$

So the Mean hydraulic depth, $\qquad m = \dfrac{A}{P} = \dfrac{7.5 \times 10^{-4}}{0.101} = 7.4 \times 10^{-3} \, \text{m}$

Manning Constant for finished concrete, $\qquad n = 0.012^{10}$ so M = 1.486/0.012=124

Flowrate, $Q = AMm^{\frac{2}{3}}\alpha^{\frac{1}{2}} = 7.5 \times 10^{-4} \times 124 \times 0.0074^{\frac{2}{3}} \times \left(\tan^{-1} 1/25\right)^{\frac{1}{2}} = 7.06 \times 10^{-4} \, \text{m}^3/\text{s}$

Or 43 liters/min or 650 gallons/hour.

Example 10.2

A drainage ditch for a local sports field consists of an excavated earth channel with a gravel lining, which gives a Manning constant, M=40. After some time the channel becomes weedy and the Manning constant is reduced to, M=33. If the ditch is semi-circular in cross-section with a diameter of 1.2m and on a gradient of 1 in 20, calculate the reduction in flowrate caused by the weeds when it is full.

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[8] www.youtube.com/watch?v=kM2XAsS4RVo
[9] www.youtube.com/watch?v=cRnIsqSTX7Q
[10] www.engineeringtoolbox.com/mannings-roughness-d_799.html

Solution:

Area of flow,
$$A = \frac{\pi d^2}{8} = \frac{\pi 1.2^2}{8} = 0.565\,\text{m}^2$$

Wetted perimeter,
$$P = \frac{\pi d}{2} + d = \frac{\pi \times 1.2}{2} + 1.2 = 3.08\,\text{m}$$

Mean hydraulic depth,
$$m = \frac{A}{P} = \frac{0.565}{3.08} = 0.183\,\text{m}$$

Difference in Flowrate:
$$dQ = (dM)Am^{\frac{2}{3}}\alpha^{\frac{1}{2}} = (40-33) \times 0.565 \times 0.183^{\frac{2}{3}} \times \left(\tan^{-1}1/20\right)^{\frac{1}{2}} = 0.284\,\text{m}^3/\text{s}$$

or 17.5% $(= (40-33)/40)$ less capacity.