EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

D5: Steady particle streams

This is an extract from 'Real Life Examples in Dynamics: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2006 (ISBN:978-0-615-20394-2) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Solids Courses. Prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Dynamics: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a junior level course in dynamics, increase the accessibility of the principles, and raise the appeal of the subject to students from diverse backgrounds. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. The 5E outline is not original and was developed by the Biological Sciences Curriculum Study[1] in the 1980s from work by Atkin and Karplus[2] in 1962. Today this approach is considered to form part of the constructivist learning theory and a number of websites provide easy-to-follow explanations of them[3].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement and enhance. Similarly, it is anticipated that these lessons plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

It is assumed that students have acquired a knowledge and understanding of topics usually found in a Sophomore level course in Statics, including free-body diagrams and efficiency.

This is the second in a series of such notes. The first in the series entitled 'Real Life Examples in Mechanics of Solids' edited by Eann Patterson (ISBN: 978-0-615-20394-2) was produced in 2006 and is available on-line at www.engineeringexamples.org.

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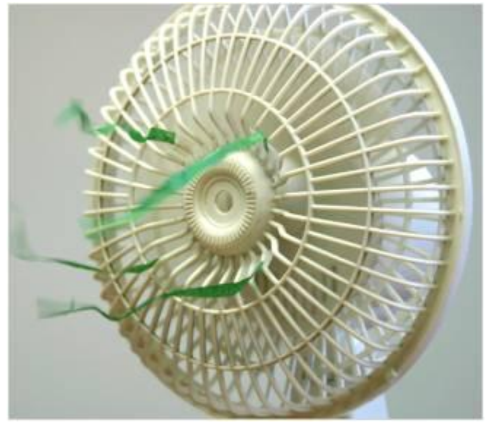
[1] Engleman, Laura (ed.), *The BSCS Story: A History of the Biological Sciences Curriculum Study*. Colorado Springs: BSCS, 2001.
[2] Atkin, J. M. and Karplus, R. (1962). Discovery or invention? *Science Teacher* 29(5): 45.
[3] e.g. Trowbridge, L.W., Bybee, R.W., *Becoming a secondary school science teacher*. Merrill Pub. Co. Inc., 1990.

KINETICS OF PARTICLES

5. Topic**: Systems of particles**



Engage:

Bring a desk fan into class, place it on the table and switch it on. It would be good to attach some ribbons or strips of tissue paper to the protective cage around the fan to illustrate the flow.

Explore:

Discuss the action of the fan blades on the air. The air on one side of the fan is essentially at rest. As it passes over the blades its momentum is increased. The blades must exert a horizontal force on the air flow in order to generate the momentum increase. Packets or bundles of air can be considered and the principle of linear impulse and momentum applied. Since the flow is steady, the applied force will be constant during a time interval.

Explain :

Explain the concept of a control volume as a 'box' around the fan containing a mass of air, *m* with an average velocity, *v*; and a small mass of air, *dm* about to enter the box with a velocity v_A at time, *t* then subsequently at time, $(t + dt)$ a corresponding mass *dm* leaves the other side of the box having been accelerated to v_B by the fan.

The principle of linear impulse and momentum:

$$\sum \int_{t_{before}}^{t_{after}} F dt = (mv)_{after} - (mv)_{before}$$

can be applied to the air stream in the box:

$$\sum F dt = ((dm \times v_B) + mv) - ((dm \times v_A) + mv)$$

i.e. $$\sum F = \frac{dm}{dt}(v_B - v_A)$$

the term dm/dt is known as the mass flow. Continuity of mass requires:

$$\frac{dm}{dt} = \rho_A v_A A_A = \rho_B v_B A_B$$

Elaborate

Fans are often powered by shaded pole AC induction motors which have a very low efficiency of the order of 20%; so a 50W fan will only deliver about 10W to the blades. Power, *P* is the rate of work, or for a fan the rate of kinetic energy change delivered to the airstream, i.e.

$$P = \frac{dKE}{dt} = \frac{\tfrac{1}{2}m\left(v_B^2 - v_A^2\right)}{dt}$$

And if the airstream is initially at rest, $v_B = 0$ then utilizing continuity of mass:

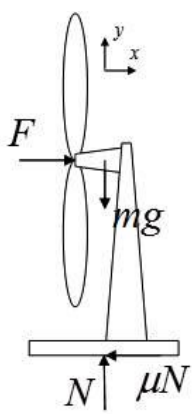
$$P = \tfrac{1}{2}\rho\left(v_A^3\right)A \qquad \text{and rearranging} \qquad v_A = \sqrt[3]{\frac{2P}{\rho A}}$$

So, if the fan has a radius of 0.1m and the density of air is 1.2 kg/m^3 then the velocity of the air leaving from the fan will be:

$$v_A = \sqrt[3]{\frac{2P}{\rho A}} = \sqrt[3]{\frac{2\times 10}{1.2\times \pi 0.1^2}} = 8.1\,\text{m/s}$$

The force on the air is $\sum F = \frac{dm}{dt}\left(v_B - v_A\right) = \rho\ A v_A^2 = 1.2\times\left(\pi 0.1^2\right)\times 8.1^2 = 2.47\,\text{N}$

Now simple statics can be employed to calculate the minimum mass of the fan to prevent it sliding along the desk:



$$\sum F_x = F - \mu N = 0 \quad \text{and} \quad \sum F_y = N - mg = 0$$

$$\text{So} \quad F = \mu mg \qquad \text{and} \qquad m = \frac{F}{\mu g} = \frac{2.47}{0.6\times 9.81} = 0.42\,\text{kg}$$

Taking the coefficient of friction between the rubber base of the fan and plastic coated table or floor to be $\mu = 0.6$.

If the fan weighs less than 420g then it will move across the floor when it is switched on.

Evaluate

Ask students to attempt the following examples:

Example 5.1

When the outlet nozzle of a 1875W hair-dryer is placed vertically above and just not touching the stainless steel plate of a set of kitchen scales, the scales register 46g when the hair-dryer is switched on. If the nozzle is an ellipse with a major diameter of 75mm and minor diameter of 22mm and the fan has a diameter of 60mm, calculate the percentage of the power used by the fan, neglecting losses.

<u>Solution</u>

Nomenclature: subscripts 'A' at the intake, 'B' at the outlet and 'C' just after impact with the plate.

The force exerted by the hairdryer can be obtained using the principle of linear impulse and momentum, and by assuming the air comes to rest (in the direction perpendicular to the plate) after impact with the plate, i.e. $v_C = 0$

$$\sum F = \frac{dm}{dt}\left(v_C - v_B\right) = \rho A_{noz} v_B^2$$

and the area of an elliptical nozzle is given by:

$$A_{noz} = \pi r_J r_N = \pi \times \frac{0.075}{2} \times \frac{0.022}{2} = 1.29 \times 10^{-3}\,\mathrm{m}$$

where r_J and r_N are the major and minor semi-axes lengths, so for air of density, ρ=1.2 kg/m^3:

$$v_B = \sqrt{\frac{F}{\rho A_{noz}}} = \sqrt{\frac{0.46 \times 9.81}{1.2 \times \left(1.29 \times 10^{-3}\right)}} = 53.9\,\mathrm{m/s}$$

For the fan, the power is the rate of delivery of kinetic energy, i.e.

$$P = \frac{d(KE)}{dt} = \frac{\tfrac{1}{2} m\left(v_B^2 - v_A^2\right)}{dt}$$

and if the airstream at the hairdryer intake is initially at rest, $v_A = 0$ then utilizing continuity of mass:

$$P = \tfrac{1}{2}\rho\left(v_B^3\right) A_{fan} = \tfrac{1}{2} \times 1.2 \times 53.9^3 \times \frac{\pi \times 0.06^2}{4} = 265\,\mathrm{W}$$

So the percentage of the power used by the fan is:

$$= \frac{265}{1875} \times 100 = 14\%$$

The remaining 86% is used in the heating elements!

<u>Example 5.2</u>

Ask the students to consider the forces acting on a supporting structure suitable for a small wind turbine capable of providing power for their home.