EVERYDAY EXAMPLES OF ENGINEERING CONCEPTS

D3: Work & Energy

This is an extract from 'Real Life Examples in Dynamics: Lesson plans and solutions' edited by Eann A. Patterson, first published in 2006 (ISBN:978-0-615-20394-2) which can be obtained on-line at www.engineeringexamples.org and contains suggested exemplars within lesson plans for Sophomore Solids Courses. Prepared as part of the NSF-supported project (#0431756) entitled: "Enhancing Diversity in the Undergraduate Mechanical Engineering Population through Curriculum Change".

INTRODUCTION

(from *'Real Life Examples in Dynamics: Lesson plans and solutions'*)

These notes are designed to enhance the teaching of a junior level course in dynamics, increase the accessibility of the principles, and raise the appeal of the subject to students from diverse backgrounds. The notes have been prepared as skeletal lesson plans using the principle of the 5Es: Engage, Explore, Explain, Elaborate and Evaluate. The 5E outline is not original and was developed by the Biological Sciences Curriculum Study[1] in the 1980s from work by Atkin and Karplus[2] in 1962. Today this approach is considered to form part of the constructivist learning theory and a number of websites provide easy-to-follow explanations of them[3].

These notes are intended to be used by instructors and are written in a style that addresses the instructor, however this is not intended to exclude students who should find the notes and examples interesting, stimulating and hopefully illuminating, particularly when their instructor is not utilizing them. In the interest of brevity and clarity of presentation, standard derivations and definitions are not included since these are readily available in textbooks which these notes are not intended to replace but rather to supplement and enhance. Similarly, it is anticipated that these lessons plans can be used to generate lectures/lessons that supplement those covering the fundamentals of each topic.

It is assumed that students have acquired a knowledge and understanding of topics usually found in a Sophomore level course in Statics, including free-body diagrams and efficiency.

This is the second in a series of such notes. The first in the series entitled 'Real Life Examples in Mechanics of Solids' edited by Eann Patterson (ISBN: 978-0-615-20394-2) was produced in 2006 and is available on-line at www.engineeringexamples.org.

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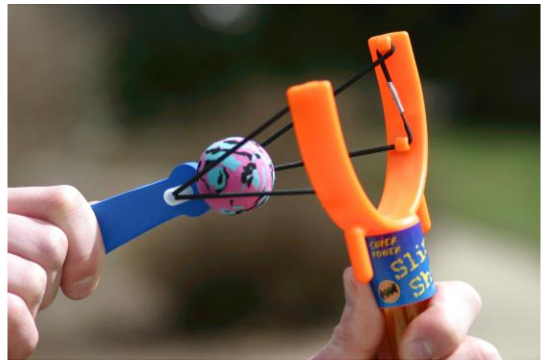
[1] Engleman, Laura (ed.), *The BSCS Story: A History of the Biological Sciences Curriculum Study*. Colorado Springs: BSCS, 2001.
[2] Atkin, J. M. and Karplus, R. (1962). Discovery or invention? *Science Teacher* 29(5): 45.
[3] e.g. Trowbridge, L.W., Bybee, R.W., *Becoming a secondary school science teacher*. Merrill Pub. Co. Inc., 1990.

KINETICS OF PARTICLES

3. Topic**: Work & Energy**



Engage:

Bring a slingshot and a handful of rubber balls into class, pull the elastic band back and release a few rubber balls into the class.

Explore:

Remind students about the basic meaning of conservation of energy. Ask students to work in pairs and to identify the conservation of energy during the loading, firing and trajectory of the balls. Invite some pairs to talk through to the class their understanding of the energy conversions. Discuss how strain energy stored in the sling material is transferred as work to the ball and becomes kinetic energy. Ask them in their pairs to reconsider conservation of energy during loading, firing and flight of projectile.

Explain:

Applying the principle of work and energy to the rubber ball during firing:

$$\sum KE_1 + \sum U_{1-2} = \sum KE_2$$

where ΣKE_1 and ΣKE_2 are the initial and final kinetic energies of the ball respectively and $\Sigma U_{1\text{-}2}$ is the work done by all the external and internal forces acting on the ball between the initial and final state.

By definition, kinetic energy, $KE = \frac{1}{2}mv^2$, and $\Sigma KE_1 = 0$ (because v_1=0).

The work done on the ball during firing is the strain energy released from the sling shot which was stored in the material as it was stretched, $U = uV$ where u is the specific strain energy at yield and V is the volume of material and assuming the sling shot is stretched to the limit of its elastic behavior.

During flight potential energy maybe gained or lost with height but this is negligible compared to the kinetic energy at launch. Some kinetic energy will be lost due to drag but again it is small so that the ball will arrive at its target with a substantial portion of the launch kinetic energy.

Elaborate

The strain energy stored at yield in an elastic band is u = 3.6 MJ/m^3; if it is of length, 150mm with a cross-section of 16mm^2 and volume, V of 2400mm^3, then the energy stored when it is pulled to yield is (see chapter 6 in Real Life Examples in Mechanics of Solids):

$$\sum U = uV = \left(3.6\times10^{6}\right)\left(2400\times10^{-9}\right) = 8.6\,\mathrm{J}$$

so for a yellow dot squash ball of mass 24g with initial kinetic energy ΣKE_1 =0:

$$\sum KE_1 + \sum U_{1-2} = \sum KE_2 = \tfrac{1}{2}mv_2^2 \text{ and } v_2 = \sqrt{\frac{2U}{m}} = \sqrt{\frac{2\times8.6}{24\times10^{-3}}} = 27\,\mathrm{ms}^{-1}$$

If the ball is launched parallel to the ground from a height of 1.5m, then its range can be calculated using kinematics (neglecting drag) since in the vertical direction: s_y = 1.5m, $v_{y=0}$ = 0, and $a_y = g$ = 9.81m/s^2,

so $\quad v_y^2 - v_{y=0}^2 = a_y s \quad$ and $\quad v_y = \sqrt{2a_y s_y} = \sqrt{2\times9.81\times1.5} = 5.4\,\mathrm{m/s}$

and $\quad v_y - v_{y=0} = 2a_y t \quad$ so $\quad t = \dfrac{v_y}{2a_y} = \dfrac{5.42}{2\times9.81} = 0.28\,\mathrm{s}$

In the horizontal direction (neglecting drag): $s = v_x t + \frac{1}{2}a_x t^2 = \left(27\times0.28\right) + 0 = 7.45\,\mathrm{m}$

so the slingshot has a range of almost 7.5m.

(Note: potential energy gained in flight is (mgh = 0.024×9.81×1.5 =) 0.35J or 4% of the kinetic energy at launch).

Evaluate

Ask students to do the following examples:

Example 3.1

A two-slice toaster is switched on by depressing a slider which causes the slices of bread to be drawn downwards into the toaster between the heating elements and also extends a spring at each end of the toaster. When the toast is ready a pair of triggers releases both springs simultaneously which in turn provide an impulse to the toast so that it pops up. A typical slice of bread is 125mm × 125mm and has a mass of 40g, and in the ‘off’ position a slice sits with two-thirds of its length inside the toaster.

(a) Calculate the force which must be applied to the slider so that toast pops up but just does not come out of the toaster when it is ready, neglecting the weight of the mechanism and assuming there are no losses in it.

(b) If the toaster is redesigned with each slot rotated outwards so that the plane of the slice is at 60° to the horizontal table top and two additional identical springs are fitted; then how far away from their respective slots will the slices land if the effect of the shape of slice is neglected.

<u>Solution</u>

(a) Toast must not jump higher than minimum depth of slot ≈ length of a slice of bread

At top of jump, vertical velocity, $v_y = v_{y2} = 0$ thus using $\quad v_{y2}^2 - v_{y1}^2 = 2a_y s_y$

$$v_{y1} = \sqrt{2a_y s_y} = \sqrt{2 \times 9.81 \times 0.125} = 1.57\,\mathrm{ms}^{-1}$$

Hence the kinetic energy received by each slice from the springs at launch is:

$$KE = \tfrac{1}{2} m v_{y1}^2 = \frac{0.04 \times 1.57^2}{2} = 0.049\,\mathrm{J}$$

To store this much energy in a spring the work done (W) on the springs is $KE = W = \tfrac{1}{2} Fd$ where F is the force applied and moved through a distance d. Since in the resting position the bread is two-thirds within the toaster the available travel for the slider, $d = \tfrac{2}{3} \times 0.125 = 0.0833\,\mathrm{m}$.

So $\quad F = \dfrac{2W}{d} = \dfrac{2 \times 0.049}{0.0833} = 1.18\,\mathrm{N}$

For the two-slice toaster the force that needs to be applied is 2.35N (= 1.18 × 2)

(b) With the additional springs the energy stored will be doubled, i.e. the kinetic energy delivered to each slice will be 0.098J (= 0.049 × 2).

Thus, the exit velocity will be $v = \sqrt{\dfrac{2 \times KE}{m}} = \sqrt{\dfrac{2 \times 0.098}{0.04}} = 2.2\ \mathrm{ms}^{-1}$ from $KE = \tfrac{1}{2} m v^2$

The vertical component will be $v_{y1} = v \sin 60 = 2.2 \sin 60 = 1.91\ \mathrm{ms}^{-1}$

and the horizontal component, $v_{x1} = v \cos 60 = 2.2 \cos 60 = 1.10\ \mathrm{ms}^{-1}$

For time to top of flight: $\quad v_{y2} - v_{y1} = a_y t$ and $v_{y2} = 0$ so $t = \dfrac{1.91}{9.81} = 0.19\,\mathrm{s}$

The time for the rise and fall to the top of the flight is 0.39 seconds (=2×0.19), and distance travelled horizontally is

$$s_x = v_x t + \tfrac{1}{2} a_x t^2 = (1.10 \times 0.39) + 0 = 0.43\,\mathrm{m}$$

i.e. each slice will land 430mm away from the toaster.

Example 3.2

Estimate the range of a longbow of the type used in medieval England (e.g. by Robin Hood). Such bows are about 2m long, made of yew with a draw of about 0.76m and were used with arrows having a mass of about 50g. The draw force has been estimated to be about 265N. To see a longbow in action just type 'longbow' in www.youtube.com (http://www.youtube.com/watch?v=YtTyOf8OCKg).

Solution:

The incremental work done, δW by the archer as the bow is pull back (drawn) over a incremental distance, δd is given by $\delta W = F\delta d$, so integrating this over the total draw gives the total work done, $W = \frac{1}{2}Fd = \frac{1}{2}\times 265\times 0.76 = 101\,\mathrm{J}$ assuming the draw force is linearly related to the draw, i.e. the bow has a constant stiffness.

This work is stored as strain energy in the bow and, when the arrow is released, it is transferred to the arrow as kinetic energy, i.e.

$$\sum KE_1 + \sum U_{1-2} = \sum KE_2 = \tfrac{1}{2}mv_2^2 \text{ and } v = \sqrt{\frac{2U}{m}} = \sqrt{\frac{2\times 101}{50\times 10^{-3}}} = 63.6\,\mathrm{ms^{-1}}$$

If the arrow is launched at an angle, α to the ground then the vertical component of velocity is $v_{y0} = v\sin\alpha$ and horizontal component is $v_{x0} = v\cos\alpha$. The time of flight can be calculated as the twice the time required to reach its maximum height when its vertical velocity, $v_y = 0$, i.e.

$$v_y - v_{y0} = a_y t \text{ and } t = \frac{-v_{y0}}{a} = \frac{v\sin\alpha}{g}$$

The distance travelled horizontally during time, $2t$ will be

$$\text{“}s = v_x t + \tfrac{1}{2}a_x t^2\text{” thus } s = (v\cos\alpha)\times 2t = \frac{2v^2}{g}\cos\alpha\sin\alpha$$

For maximum range $\frac{ds}{d\alpha} = 0 = \frac{2v^2}{g}\left(\cos^2\alpha - \sin^2\alpha\right)$ i.e. $\frac{\sin\alpha}{\cos\alpha} = 1$ and $\alpha = 45°$

Hence substituting in $s = \frac{2v^2}{g}\cos\alpha\sin\alpha = \frac{2\times 63.6^2}{9.81}\times\left(\frac{1}{\sqrt{2}}\right)^2 = 412\,\mathrm{m}$

Replica longbows have been shown to have a range of about 300m so this estimate is a little long which is unsurprising since drag and energy losses were neglected.